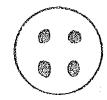
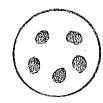
1. Take 12 pebbles. Make 3 piles like this.

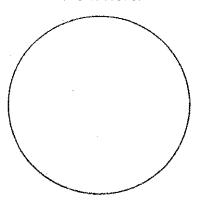


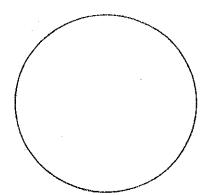


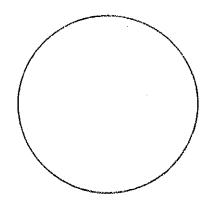


Now move 1 pebble so that each pile has 4 pebbles.

Do it here.



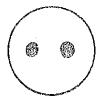




2. Take 12 pebbles. Make 3 piles like this.







This time you have 2 moves. On the first, move 1 pebble. On the second, move 2 pebbles from 1 pile to another. Try to rearrange the pebbles into 3 equal piles. Use the 3 circles above.

3. Now use the same rules and try these combinations.

a)





b)

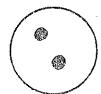




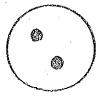


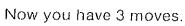
c)





Take 12 pebbles. Make piles of 2, 4, and 6 pebbles like this.







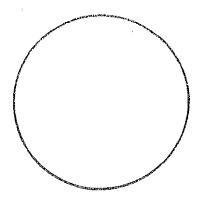


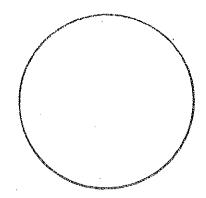
First: Move 1 pebble.

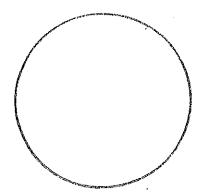
Second: Move 2 pebbles.

Third: Move 3 pebbles.

Try and make 3 piles of 4 pebbles after the 3 moves.







Take 12 pebbles. Start with piles of:

- 1. 1, 3, 8
- 2. 3, 4, 5
- 3. 1, 10, 1
- 4. 0, 3, 9
- 5. 2, 8, 2
- 6. 3, 6, 3
- 7. 1, 4, 7
- 8. 0, 5, 7

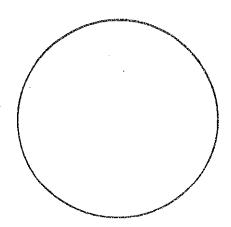
You have 3 moves.

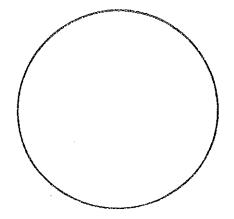
First: Move 1 pebble.

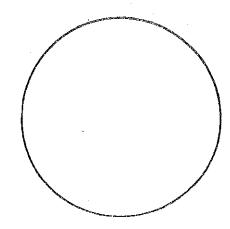
Second: Move 2 pebbles.

Third: Move 3 pebbles.

Try and make 3 piles of 4 pebbles after the 3 moves.

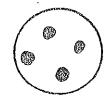






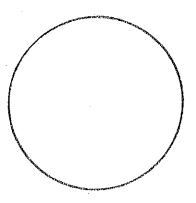
1. Start with 3 piles of 4 pebbles each.

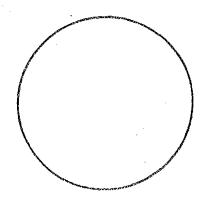


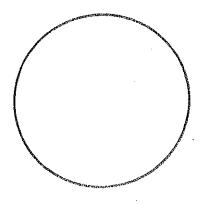




Move 1 pebble on the first move. Move 2 pebbles on the second move, 3 on the third, and so on. Can you go on for 12 moves and end up with 12 pebbles in 1 pile and no other piles?



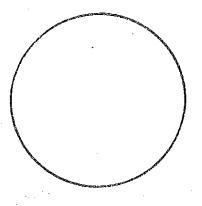


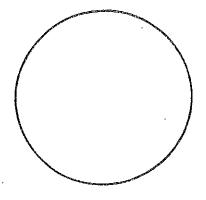


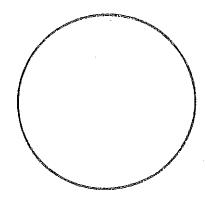
2. Start with 3 piles of 4 pebbles each. Use the same moves as above. What is the fewest number of moves it takes to get the 12 pebbles in 1 pile and no other piles?

3. Start with a pile of 12 pebbles and no other piles.

Can you get back to 3 piles of 4, 4, and 4 pebbles moving 1 pebble the first time, 2 pebbles the second, and so on?







Make 3 piles with 4 pebbles in each pile.

On each move a player must move pebbles from only 1 pile to another.

On the first move, the first player must move 1 pebble.

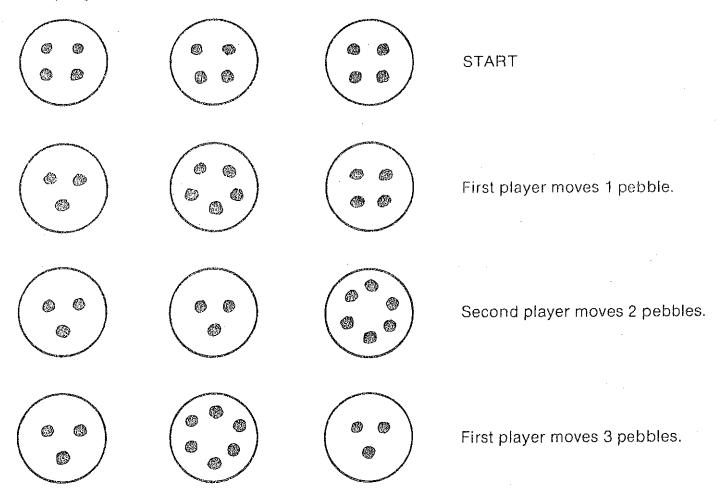
On the second move, the second player must move 2 pebbles.

On the third move, the first player must move 3 pebbles.

On the fourth move, the second player must move 4 pebbles, and so on.

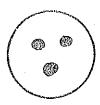
The first player who cannot move loses the game.

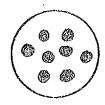
## Sample game:

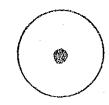


Game continues until 1 player cannot make a move with the correct number of pebbles.

1. Take 12 pebbles. Start with them in piles of 3, 8, and 1, like this.







On each move you may add to a pile the number of pebbles already in that pile. For example, you could take 3 from the pile of 8 and add them to the pile of 3. Or you could take 1 from the pile of 3, or 8, and add it to the pile of 1.







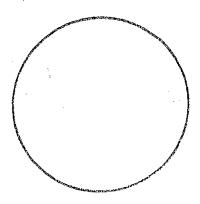
or

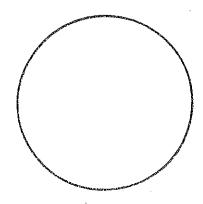


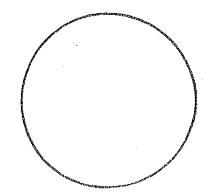




Try to make 3 piles of 4 pebbles each in 3 moves.



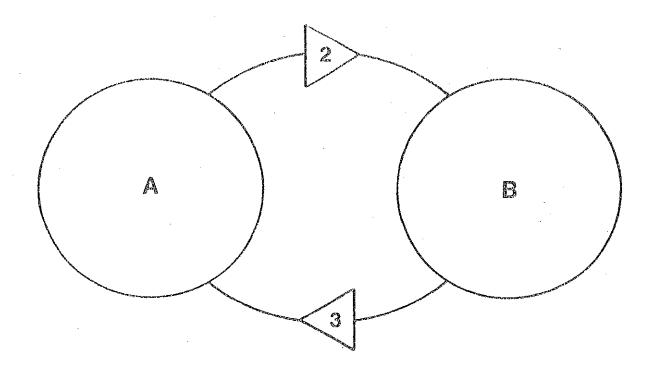




- 2. Now do the same thing, starting with piles of:
  - a) 2, 9, 1
  - b) 2, 7, 3
- 3. Do the above activities again moving different numbers of pebbles each move.

Can each of them be done in a different way?

Can you do 2b in just 2 moves?



- 1. Put 5 pebbles in circle A. Can you get all the pebbles from A to B? You may move pebbles in groups of 2 along the 2 route and in groups of 3 along the 3 route in the directions shown by the arrows.
- 2. Now start with 7 pebbles in A. How many moves does it take to get all the pebbles to B?
- 3. Start with 5 pebbles in circle B. Can you get them all to A? What is the fewest number of moves it takes?
- 4. Start with 7 pebbles in B. Move them all to A. What is the fewest number of moves it takes?